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String theoretic QCD axion with stabilized saxion and the pattern of supersymmetry breaking

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Abstract: String theoretic axion is a prime candidate for the QCD axion solving the strong CP problem. For a successful realization of the QCD axion in string theory, one needs to stabilize moduli including the scalar partner (saxion) of the QCD axion, while keeping the QCD axion unfixed until the low energy QCD instanton effects are turned on. We note that a simple generalization of KKLT moduli stabilization provides such set-up realizing the axion solution to the strong CP problem. Although some details of moduli stabilization are different from the original KKLT scenario, this set-up leads to the mirage mediation pattern of soft SUSY breaking terms as in the KKLT case, preserving flavor and CP as a consequence of approximate scaling and axionic shift symmetries. The set-up also gives an interesting pattern of moduli masses which might avoid the cosmological moduli, gravitino and axion problems.

Keywords: Axion, Moduli Stabilization, Supersymmetry Breaking.

Contents

1. Introduction

The strong CP problem [1] is a naturalness problem arising from that CP is conserved by the strong interactions but not by the weak interactions. The low energy QCD lagrangian contains a CP violating angle $\bar{\theta} = \theta_{QCD} + \text{argDet}(\lambda_u \lambda_d) + ...$, where θ_{QCD} is the bare QCD vacuum angle, $\lambda_{u,d}$ are the Yukawa couplings of the up and down-type quarks, and the ellipses stands for the contribution from other high energy parameters, e.g. the gluino mass and B-parameter for the case of supersymmetric models. The observed CP violations in K and B meson system suggest that $\lambda_{u,d}$ are complex parameters with phases of order unity. On the other hand, the non-observation of the neutron electric dipole moment implies that $|\bar{\theta}| \lesssim 10^{-9}$. This raises the question why the phase combination $\bar{\theta}$ is so small.

There are presently three known solutions to the strong CP problem. One simple possibility is that the up quark is massless, rendering the CP violations from $\bar{\theta}$ vanish. A massless up quark might not be in conflict with the known low energy properties of QCD since an effective up quark mass can be mimicked by instanton effects [2]. A second solution is that CP is an exact symmetry of the underlying fundamental theory [3], but is broken spontaneously in a specific manner to give $\bar{\theta}$ small enough [4]. The third solution is to have a non-linearly realized global $U(1)_{PQ}$ symmetry which is explicitly broken by the QCD anomaly [5]. This solution predicts a light pseudo Goldston boson, the axion [6], which might have interesting cosmological and/or astrophysical implications [1].

Compactified string theory contains numerous axions which originate from higher-dimensional antisymmetric tensor fields [7], thus is perhaps the most plausible framework to give the QCD axion solving the strong CP problem [8, 9, 10, 11]. If we assume supersymmetric compactification, axions are accompanied by their scalar partners. In this case, much of the physical properties of the QCD axion depends on the mechanism stabilizing its scalar partner "saxion". For instance, the axion decay constant and the strength of unwanted (non-perturbative) $U(1)_{PQ}$ breaking other than the QCD anomaly can be determined only after the saxion vacuum value is fixed. Axion cosmology is another subject depending severely on the saxion stabilization mechanism. Thus an explicit realization of saxion stabilization is mandatory in order to see if a specific string compactification can successfully realize the axion solution to the strong CP problem.

A key requirement for the saxion stabilization is that it should keep the QCD axion as a flat direction until the low energy QCD instanton effects are taken into account. Unless the QCD axion mass from saxion stabilization is extremely suppressed as Eq. (2.3), the dynamical relaxation of $\bar{\theta}$ can not be accomplished. In light of the recent progress in moduli stabilization [12], an immediate step toward string theoretic QCD axion would be $U(1)_{PQ}$-invariant generalization of KKLT moduli stabilization which starts with supersymmetric AdS solution lifted later to dS (or Minkowski) vacuum by an uplifting potential [13]. In regard to this possibility, it has been noticed recently that supersymmetric solution of any $U(1)_{PQ}$-invariant effective SUGRA gives a tachyonic saxion mass [11]. This might be considered as an indication that QCD axion favors non-supersymmetric moduli stabilization such as the perturbative stabilization discussed in [14] or the large volume stabilization advocated in [15].

In this paper, we point out that an uplifting potential induced by SUSY breaking brane stabilized at the end of warped throat [13, 16, 17], which is in fact the most plausible form of uplifting potential in KKLT-type compactification, automatically solves the tachyonic saxion problem for $U(1)_{PQ}$-invariant generalization of KKLT moduli stabilization with the number of Kähler moduli $h_{1,1} > 1$. We also examine the pattern of moduli masses and soft SUSY breaking terms of visible fields in this set-up giving the QCD axion.

Quite interestingly, although some details of moduli stabilization are different from the original KKLT scenario, the resulting soft SUSY breaking terms still receive comparable contributions from moduli mediation (including the saxion mediation) [18] and anomaly mediation [19], thereby take the mirage mediation pattern [16, 20, 21, 22, 23, 24] as in the KKLT case*. Furthermore, the soft terms naturally preserve flavor and CP as a consequence of approximate scaling and axionic-shift symmetries of the underlying string compactification, independently of the detailed forms of moduli Kähler potential and matter Kähler metric. As for the flavor conservation, the universality of moduli (including the saxion) F-components which is another interesting feature of our set-up plays an important role.

Our moduli stabilization set-up gives also an interesting pattern of moduli masses. Independently of the detailed form of the moduli Kähler potential, saxion has a mass $m_s \simeq \sqrt{2} m_{3/2}$, while the other Kähler moduli (except for the QCD axion) have a mass of the order of $m_{3/2} \ln(M_{Pl}/m_{3/2})$, and the visible sector superparticles have soft masses of the order of $\frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}$. If the visible sector superparticles are assumed to have the weak scale masses, the saxion has a right mass to decay right before the big-bang nucleosynthesis (BBN), while the other moduli are heavy enough to decay well before the BBN. This feature leads to moduli cosmology different from the original KKLT set-up [26], and might allow to avoid the cosmological gravitino, moduli and axion problems [27]. In particular, it might allow the QCD axion to be a good dark matter candidate under a mild assumption on the initial axion misalignment although the axion decay constant is near the GUT scale.

*We note that our saxion stabilization scheme does not give the pattern of soft terms proposed in [25].

2. Saxion stabilization

Let T denote the modulus superfield whose pseudoscalar component $\text{Im}(T)$ corresponds to the QCD axion solving the strong CP problem. For the dynamical relaxation of $\bar{\theta}$, $\text{Im}(T)$ is required to couple to the QCD anomaly $F\tilde{F}$, i.e. the holomorphic gauge kinetic function of QCD should depend on T as

$$f_a = c_T T + \Delta f_a(\Phi^i), \tag{2.1}$$

where c_T is a real nonzero constant, and Φ^i are generic moduli other than T. To avoid saxion-mediated macroscopic force, the saxion $s = \sqrt{2}\text{Re}(T)$ should be stabilized with $m_s \gtrsim 10^{-3}$ eV. In fact, cosmological consideration typically requires much heavier saxion mass, e.g. $m_s \gtrsim 40$ TeV for the saxion decay before the BBN in case that saxion couplings are Planck-scale suppressed [28]. On the other hand, in order to keep the dynamical relaxation of $\bar{\theta}$ available, the axion $a = \sqrt{2}\text{Im}(T)$ should remain to be unfixed until the low energy QCD instanton effects are taken into account. More explicitly, saxion stabilization mechanism should preserve the non-linear PQ symmetry

$$U(1)_{PQ}: \ T \rightarrow T + i\beta \quad (\beta = \text{real constant}) \tag{2.2}$$

to the accuracy that axion mass induced by the saxion stabilization is small as

$$\delta m_a \lesssim \frac{\sqrt{10^{-9} m_\pi^2 f_\pi^2}}{v_{\rm PQ}} \sim 10^{-6} \left(\frac{10^9 \text{ GeV}}{v_{\rm PQ}}\right) \text{eV}, \tag{2.3}$$

where $v_{\rm PQ}$ is the axion decay constant which is constrained to be bigger than 10^9 GeV [1].

The above condition strongly suggests that saxion should be stabilized within the framework of $U(1)_{PQ}$-invariant effective SUGRA in which the Kähler potential and superpotential take the form

$$K = K(T + T^*, \Phi^i, \Phi^{i*}), \quad W = W(\Phi^i). \tag{2.4}$$

Note that a T-dependent superpotential generically breaks $U(1)_{PQ}$[†], giving an axion mass comparable to the saxion mass. A simple example of $U(1)_{PQ}$-invariant saxion stabilization has been discussed before [14] within the framework of flux compactification of type IIB string theory. In [14], it was assumed that the IIB dilaton and complex structure moduli are stabilized by 3-form fluxes, leaving the single Kähler modulus T unfixed. If the visible gauge fields live on $D7$ branes wrapping the 4-cycle whose volume corresponds to $\text{Re}(T)$, the QCD gauge kinetic function is given by $f_a = T$, thereby $\text{Im}(T)$ can be a candidate for the QCD axion. At leading order, the Kähler potential of T takes the no-scale form. However at higher order in α' and string loop expansion, it receives $U(1)_{PQ}$-invariant corrections as

$$K = -3\ln(T + T^*) + \frac{\xi_1}{(T + T^*)^{3/2}} - \frac{\xi_2}{(T + T^*)^2}, \tag{2.5}$$

[†] A superpotential of the form $W = e^{-bT}\Omega(\Phi^i)$ (b = real constant) preserves $U(1)_{PQ}$. However such form of superpotential typically leads to a runaway of saxion unless an uncontrollably large quantum correction to Kähler potential is assumed [30]. Here we are interested in the possibility to stabilize the saxion in a region of moduli space where the leading order Kähler potential is reliable.

where ξ_1 is the coefficient of higher order α' correction which is positive for a positive Euler number, and ξ_2 is the coefficient of string loop correction. Assuming a flux-induced constant superpotential $W = w_0$, the resulting scalar potential stabilizes $\mathrm{Re}(T)$ while keeping $\mathrm{Im}(T)$ unfixed if $\xi_1 > 0$ and $\xi_2 > 0$. In this scenario, saxion is stabilized by the competition between two controllably small perturbative corrections, which is possible because the potential is flat in the limit $\xi_1 = \xi_2 = 0$.

Although attractive, the above saxion stabilization can be applied only for $\xi_1 > 0$ which requires the Euler number $\chi = 2(h_{1,1} - h_{2,1}) > 0$. On the other hand, most of interesting Calabi-Yau (CY) compactifications have nonzero $h_{2,1}$. In particular, if one wishes to get a landscape of flux vacua which might contain a state with nearly vanishing cosmological constant [29], one typically needs large number of 3-cycles, e.g. $h_{2,1} = \mathcal{O}(100)$, to accommodate 3-form fluxes [12]. In such case, there remain (many) Kähler moduli not stabilized by the above purely perturbative mechanism.

A simple way out of this difficulty would be to stabilize all Kähler moduli other than saxion by non-perturbative superpotential a la KKLT, while keeping the saxion stabilized by $U(1)_{PQ}$-invariant Kähler potential. In this case, the saxion potential resulting from the leading order Kähler potential is *not* flat anymore, therefore saxion can not be stabilized by controllably small perturbative corrections to the Kähler potential. Still this kind of generalized KKLT set-up might allow a supersymmetric solution of

$$D_I W = \partial_I W + W \partial_I K = 0 \tag{2.6}$$

in a region of moduli space where the leading order Kähler potential is reliable. To see that this is a rather plausible possibility, let us consider a simple example with

$$\begin{aligned} K &= -2\ln\left[(T_1 + T_1^*)^{3/2} - (T_2 + T_2^*)^{3/2} - (T_3 + T_3^*)^{3/2}\right], \\ W &= w_0 + A_1 e^{-b_1 T_1} + A_2 e^{-b_2(T_2+T_3)}, \end{aligned} \tag{2.7}$$

where $w_0 \sim m_{3/2}$ and $A_{1,2} \sim 1$ in the unit with $M_{Pl} = 1$. (Unless specified, we use the unit with $M_{Pl} = 1$ throughout this paper.) Here $K = -2\ln(V_{CY})$ corresponds to the leading order Kähler potential of the Kähler moduli T_i for a CY volume given by

$$V_{CY} = \int J \wedge J \wedge J = t_1^3 - t_2^3 - t_3^3, \tag{2.8}$$

where J is the Kähler two form and $3(T_i + T_i^*) = \partial V_{CY}/\partial t_i$. For the above non-perturbative superpotential, it is convenient to define new chiral superfields as

$$\Phi_1 = T_1, \quad \Phi_2 = T_2 + T_3, \quad T = T_2 - T_3, \tag{2.9}$$

where T corresponds to the invariant direction of W. It is then straightforward to see that the model allows a SUSY solution:

$$\begin{aligned} T_1 &\simeq \frac{1}{b_1}\ln(M_{Pl}/m_{3/2}), \\ T_2 = T_3 &\simeq \frac{1}{2b_2}\ln(M_{Pl}/m_{3/2}), \end{aligned} \tag{2.10}$$

for which the leading order Kähler potential is a good approximation if $m_{3/2}$ is hierarchically lighter than M_{Pl}.

Since it is always a stationary point of the scalar potential

$$V_F = e^K \left(K^{I\bar{J}} D_I W (D_J W)^* - 3|W|^2 \right), \tag{2.11}$$

supersymmetric moduli configuration is a good starting point for moduli stabilization. On the other hand, it has been noticed that supersymmetric moduli stabilization in $U(1)_{PQ}$-invariant effective SUGRA (2.4) always gives a tachyonic saxion [11]. For $D_I W = 0$ with $W \neq 0$, one easily finds

$$\left(\frac{\partial^2 V_{\rm F}}{\partial s^2} \right)_{D_I W = 0} = -2|m_{3/2}|^2 \partial_T \partial_{\bar{T}} K < 0, \tag{2.12}$$

where $s = \sqrt{2}{\rm Re}(T)$ is the saxion and $m_{3/2} = e^{K/2} W$ is the gravitino mass, thus there is a tachyonic direction which has a nonzero mixing with the saxion field. However supersymmetric moduli configuration generically gives an AdS vacuum, thus requires an uplifting potential in order to be a phenomenologically viable dS (or Minkowski) vacuum. A simple way to get uplifting is to introduce SUSY breaking brane carrying a positive tension as in KKLT [13]. If the underlying geometry has a warped throat [31, 32], the SUSY breaking brane is stabilized at the end of throat [13] independently of the details of SUSY breaking dynamics. In the following, we show that the uplifting potential induced by SUSY breaking brane stabilized at the end of warped throat, which is the most natural form of uplifting potential in KKLT-type moduli stabilization, automatically solves the tachyonic saxion problem.

To this end, let us consider the KKLT-type compactification of Type IIB string theory on CY orientifold with the number of Kähler moduli $h_{1,1} > 1$ and the visible gauge fields living on $D7$ branes[‡]. As usual, we assume that the string dilaton and all complex structure moduli are fixed by 3-form fluxes with masses hierarchically heavier than the Kähler moduli and gravitino masses, and consider the effective SUGRA of Kähler moduli

$$\Phi^I \ = \ (T, \Phi^i) = \frac{1}{\sqrt{2}} (s + ia, \phi^i + ia^i), \tag{2.13}$$

where ${\rm Re}(\Phi^I)$ and ${\rm Im}(\Phi^I)$ correspond to appropriate linear combinations of the 4-cycle volumes and the RR 4-form axions, respectively. If ${\rm Re}(T)$ and Φ^i can be stabilized while preserving the non-linear $U(1)_{PQ}$ symmetry (2.2), ${\rm Im}(T)$ can play the role of QCD axion. Following KKLT, we also assume that Φ^i are stabilized by non-perturbative effects such as $D3$-instanton or a gaugino condensation on the hidden $D7$-branes warpping certain 4-cycles in CY. Finally, we introduce a SUSY breaking brane without specifying the SUSY breaking dynamics, which will be stabilized at the end of (maximally) warped throat independently of the detailed SUSY breaking dynamics.

[‡]If the visible gauge fields live on $D3$, the corresponding gauge kinetic function $f_a = S$ can not give the QCD axion since the IIB dilaton S is fixed by the flux-induced superpotential $W_{\rm flux} = \int \Omega \wedge (F_3 - iSH_3)$.

After integrating out the heavy dilaton and complex structure moduli as well as the SUSY-breaking fields on the uplifting brane, the effective action of Kähler moduli can be written as [16]

$$\int d^4\theta CC^*\Big[-3e^{-K/3} - CC^*\Lambda^2\bar{\Lambda}^2 e^{4A}\mathcal{P}\Big] + \left[\left[\int d^2\theta C^3\Big(w_0 + \sum_i A_i e^{-b_i\Phi^i}\Big) + \text{h.c.}\right]\right], \tag{2.14}$$

where K is the effective Kähler potential of Kähler moduli $\Phi^I = (T, \Phi^i)$, C is the chiral compensator superfield of 4D $N = 1$ SUGRA, e^{4A} is the red-shift factor [31] of the effective Volkov-Akulov action of the Goldstino superfield,

$$\Lambda^\alpha = \theta^\alpha + \text{Goldstino-dependent terms}, \tag{2.15}$$

which is localized on SUSY-breaking brane. This Volkov-Akulov action provides a low energy description of generic SUSY-breaking brane stabilized at the end of warped throat, e.g. anti-$D3$ brane or any brane which carries 4D dynamics breaking $N = 1$ SUSY spontaneously[§]. Here the compensator dependence of the Volkov-Akulov term is determined by that it corresponds to an uplifting potential with mass-dimension four in the unitary gauge $\Lambda^\alpha = \theta^\alpha$, and the warp factor $e^{4A} \ll 1$ is determined by the complex structure modulus of the collapsing 3-cycle [32]. The axionic shift symmetries of the RR 4-form axions $\text{Im}(\Phi^i)$,

$$U(1)_i:\ \Phi^i \to \Phi^i + i\beta_i \quad (\beta_i = \text{real constants}), \tag{2.16}$$

are assumed to be broken by $D3$ instantons and/or hidden $D7$ gaugino condensations generating the non-perturbative superpotential $\sum_i A_i e^{-b_i\Phi^i}$, while the axionic shift symmetry (2.2) for the QCD axion $\text{Im}(T)$ is preserved at this stage. In this scheme, possible non-perturbative $U(1)_i$-breaking in Kähler potential can be ignored, and then K takes the form:

$$K = K(T + T^*, \Phi^i + \Phi^{i*}). \tag{2.17}$$

Note that the chiral superfields Φ^i are defined through the exponents of non-perturbative superpotential. The constants w_0 and A_i in the superpotential can be always made to be real by appropriate $U(1)_R$ and $U(1)_i$ transformations, which will be crucial for the soft SUSY breaking terms to preserve CP [37].

An important feature of KKLT-type compactification is that in the limit $e^{4A} \ll 1/V_{CY}^{2/3}$, the uplifting brane is separated from CY by a strongly warped throat: CY corresponds to the UV end of warped throat, while SUSY-breaking brane is located at the IR end (See Fig. 1.). In this case, Kähler moduli on CY and Goldstino superfield on SUSY breaking brane are sequestered from each other [16, 38, 39], i.e. $\mathcal{P}$ is independent of Φ^I. Indeed, a full 10-dimensional analysis gives [40]

$$\mathcal{P} = \mathcal{P}_0\Big[1 + \mathcal{O}(e^{4A}V_{CY}^{2/3})\Big] \tag{2.18}$$

[§]For more detailed discussion of this point, see [33, 24, 34, 35, 36].

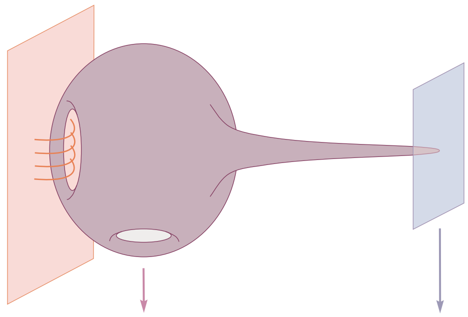

Figure 1: CY space and SUSY-breaking brane sequestered from each other by warped throat. Here the visible sector $D7$ branes are assumed to be wrapping a 4-cycle of CY.

for $e^{4A} \ll 1/V_{CY}^{2/3}$, where $\mathcal{P}_0$ is a Φ^I-independent constant[¶]. Note that the visible sector $D7$ branes wrap a 4-cycle in CY at the UV end of throat, thus we can realize the conventional high scale gauge coupling unification even in the presence of highly warped throat.

From the above discussion, one finds that generic SUSY-breaking brane stabilized at the IR end of warped throat provides a *sequestered* form of Volkov-Akulov operator in $N = 1$ superspace:

$$C^2 C^{*2} \Lambda^2 \bar{\Lambda}^2 e^{4A} \mathcal{P}_0 = C^2 C^{*2} \theta^2 \bar{\theta}^2 e^{4A} \mathcal{P}_0 + \text{Goldstino dependent terms}, \tag{2.19}$$

where $e^{4A}\mathcal{P}_0$ is a constant. In the Einstein frame with $C = e^{K/6}$, this Volkov-Akulov operator adds an uplifting potential $V_{\rm lift} = e^{4A}\mathcal{P}_0 e^{2K/3}$ to the conventional F-term potential V_F, thereby the total moduli potential is given by

$$V_{\rm TOT} = V_F + V_{\rm lift} = e^K \left(K^{I\bar{J}} D_I W (D_J W)^* - 3|W|^2 \right) + e^{4A}\mathcal{P}_0 e^{2K/3}. \tag{2.20}$$

On the other hand, the Volkov-Akulov operator does not affect the on-shell expression of the moduli F-components, thus F^I in the Einstein frame takes the conventional form:

$$F^I = -e^{K/2} K^{I\bar{J}} (D_J W)^*. \tag{2.21}$$

Obviously, the uplifting potential $V_{\rm lift} = e^{4A}\mathcal{P}_0 e^{2K/3}$ gives a positive saxion mass-square for supersymmetric moduli configuration

$$\left(\frac{\partial^2 V_{\rm lift}}{\partial s^2} \right)_{D_I W = 0} = \frac{4}{3} V_{\rm lift} \partial_T \partial_{\bar{T}} K \simeq 4|m_{3/2}|^2 \partial_T \partial_{\bar{T}} K, \tag{2.22}$$

where we have used the SUSY condition $\partial_T K = 0$ and also the condition of vanishing cosmological constant:

$$\left(V_{\rm lift} \right)_{D_I W = 0} \simeq -\left(V_F \right)_{D_I W = 0} \simeq 3|m_{3/2}|^2. \tag{2.23}$$

[¶]Schematically, $\mathcal{P}$ depends on the Kähler moduli as $\mathcal{P} \propto 1/(1 + e^{4A} V_{CY}^{2/3})$ [40], which gives $\mathcal{P} = \mathcal{P}_0 V_{CY}^{-2/3} [1 + \mathcal{O}(1/e^{4A} V_{CY}^{2/3})]$ in the opposite limit of large volume and weakly warped throat with $1/V_{CY}^{2/3} \ll e^{4A}$.

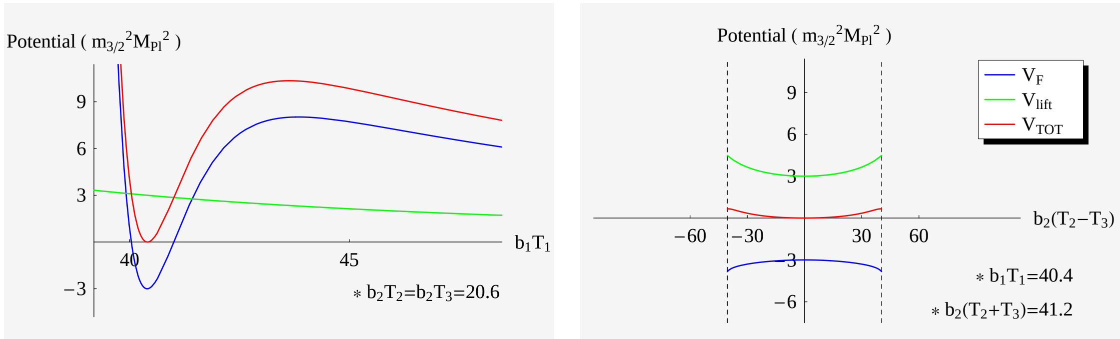

Figure 2: Moduli potential along T_1 and $T = T_2 - T_3$ in unit of $m_{3/2}^2 M_{Pl}^2$ for the toy model (2.7). T_1 is stabilized by the KKLT superpotential giving $m_{T_1} \sim m_{3/2} \ln(M_{Pl}/m_{3/2})$, while the saxion $\text{Re}(T)$ is stabilized with $m_s \simeq \sqrt{2} m_{3/2}$ by the sequestered uplifting potential.

This positive saxion mass-square from $V_{\rm lift}$ always dominates over the tachyonic saxion mass-square from V_F, thereby stabilizing the saxion as

$$\left(\frac{\partial^2 V_{\rm TOT}}{\partial s^2} \right)_{D_I W = 0} \simeq 2|m_{3/2}|^2 \partial_T \partial_{\bar{T}} K. \tag{2.24}$$

Schematically, Φ^i are stabilized by the KKLT-type of superpotential, while the saxion is stabilized by $V_{\rm lift}$. In this procedure, it is essential to have non-zero Kähler mixing between Φ^i and T, i.e. $\partial_i \partial_{\bar{T}} K \neq 0$, as it allows $\partial_T K = 0$ in a region where the leading order Kähler potential is reliable. Since $\text{Re}(T)$ is a linear combination of 4-cycle volumes which corresponds to the invariant direction of non-perturbative superpotential, while $\text{Re}(\Phi^i)$ are other combinations corresponding to the exponents of non-perturbative superpotential, such Kähler mixing between T and Φ^i is a generic feature of the moduli Kähler potential.

In Fig. 2, we show the behaviors of V_F, $V_{\rm lift}$ and $V_{\rm TOT}$ along the KKLT Kähler modulus $\text{Re}(T_1)$ and the saxion $\text{Re}(T) = \text{Re}(T_2 - T_3)$ for the toy example (2.7). Note that all 4-cycle volumes $\text{Re}(T_I)$ $(I = 1, 2, 3)$ have large positive vacuum values in the limit $m_{3/2} \ll M_{Pl}$, justifying the leading order Kähler potential in (2.7). The vanishing saxion vacuum value, $\langle \text{Re}(T) \rangle = 0$, is a result of convention, and does not cause any trouble. In the next section, we provide a more detailed analysis of the moduli masses and the pattern of SUSY breaking F-components in generic $U(1)_{PQ}$-invariant generalization of KKLT set-up.

3. Moduli masses, F-components and the axion scale

To examine the moduli masses and the pattern of SUSY breaking F-components, let us expand the total moduli potential $V_{\rm TOT} = V_F + V_{\rm lift}$ and the moduli F-components F^I around the supersymmetric moduli configuration $\Phi^I_0 = (T_0, \Phi^i_0)$ satisfying

$$D_i W(\vec{\Phi}_0) = 0, \quad D_T W(\vec{\Phi}_0) = W(\vec{\Phi}_0) \partial_T K(\vec{\Phi}_0) = 0. \tag{3.1}$$

We then find

$$V_{\rm TOT} = -3|m_{3/2}(\vec{\Phi}_0)|^2 + V_{\rm lift}(\vec{\Phi}_0) + \sqrt{2} \delta\phi^I \partial_I V_{\rm lift}(\vec{\Phi}_0)$$

$$
\begin{aligned}
&+\ \frac{1}{4}\delta\phi^I\delta\phi^J(\partial_I\partial_{\bar{J}}+\partial_{\bar{I}}\partial_J+\partial_I\partial_J+\partial_{\bar{I}}\partial_{\bar{J}})V_{\rm TOT}\\
&+\ \frac{1}{4}\delta a^I\delta a^J(\partial_I\partial_{\bar{J}}+\partial_{\bar{I}}\partial_J-\partial_I\partial_J-\partial_{\bar{I}}\partial_{\bar{J}})V_{\rm TOT}+\mathcal{O}((\delta\Phi)^3),\\
&=\ -3|m_{3/2}(\vec{\Phi}_0)|^2+V_{\rm lift}(\vec{\Phi}_0)+\frac{2\sqrt{2}}{3}\partial_I K(\vec{\Phi}_0)V_{\rm lift}(\vec{\Phi}_0)\delta\phi^I\\
&+\ \frac{1}{2}\left(m_\phi^2\right)_{IJ}\delta\phi^I\delta\phi^J+\frac{1}{2}\left(m_a^2\right)_{IJ}\delta a^I\delta a^J+\mathcal{O}((\delta\Phi)^3),\\
F^I &=\ -\frac{1}{\sqrt{2}}m_{3/2}^*K^{I\bar{J}}\Big[(G_{\bar{J}\bar{L}}+K_{\bar{J}L})\delta\phi^L-i(G_{\bar{J}\bar{L}}-K_{\bar{J}L})\delta a^L\Big],
\end{aligned}
\tag{3.2}
$$

where

$$
\Phi^I-\Phi_0^I\equiv\frac{1}{\sqrt{2}}(\delta\phi^I+i\delta a^I)=\frac{1}{\sqrt{2}}(\delta s+i\delta a,\delta\phi^i+i\delta a^i)
\tag{3.3}
$$

are the moduli and axion fluctuations from the supersymmetric configuration $\vec{\Phi}_0$, and

$$
\begin{aligned}
&K_{I\bar{J}}=\partial_I\partial_{\bar{J}}K,\quad G_{IJ}=\partial_I\partial_J(K+\ln|W|^2),\\
&\left(m_\phi^2\right)_{IJ}=|m_{3/2}|^2\left(G_{IL}G_{\bar{J}\bar{M}}K^{L\bar{M}}-G_{IJ}-2K_{I\bar{J}}\right)\\
&\qquad\qquad+\frac{4}{3}\left(K_{I\bar{J}}+\frac{2}{3}\partial_I K\partial_{\bar{J}}K\right)V_{\rm lift},\\
&\left(m_a^2\right)_{IJ}=|m_{3/2}|^2(G_{IL}G_{\bar{J}\bar{M}}K^{L\bar{M}}+G_{IJ}-2K_{I\bar{J}})
\end{aligned}
\tag{3.4}
$$

for $\partial_I=\partial/\partial\Phi^I$ and $\partial_{\bar{I}}=\partial/\partial\Phi^{I*}$. Here we are using that $K=K(\Phi^I+\Phi^{I*})$, $V_{\rm lift}(\Phi^I+\Phi^{I*})$, $\vec{\Phi}_0$ is a stationary point of V_F, and the effective SUGRA (2.14) preserves CP as a consequence of $U(1)_i$ and $U(1)_{PQ}$ which assure that w_0 and A_i in W can be chosen to be real and all derivatives of K and $V_{\rm lift}$ are automatically real [37]. The kinetic terms of moduli and axion fluctuations are given by

$$
\mathcal{L}_{\rm kin}=\frac{1}{2}K_{I\bar{J}}(\vec{\Phi}_0)\Big[\partial_\mu\delta\phi^I\partial^\mu\delta\phi^J+\partial_\mu\delta a^I\partial^\mu\delta a^J\Big].
\tag{3.5}
$$

The SUSY condition $D_iW=0$ gives

$$
\Phi_0^i=\frac{1}{b_i}\ln\left(\frac{A_i(b_i-\partial_iK)}{w_0\partial_iK}\right)=\frac{\ln(M_{Pl}/m_{3/2})}{b_i}\left[1+\mathcal{O}\left(\frac{1}{\ln(M_{Pl}/m_{3/2})}\right)\right],
\tag{3.6}
$$

where we assumed that A_i are of the order of unity, while $w_0\sim m_{3/2}$ is hierarchically small to get the low energy SUSY at final stage[‖]. In fact, the little hierarchy factor $\ln(M_{Pl}/m_{3/2})$ allows a perturbative expansion in powers of

$$
\epsilon\equiv\frac{1}{\ln(M_{Pl}/m_{3/2})}\sim\frac{1}{4\pi^2}.
\tag{3.7}
$$

[‖]Such a small w_0 might be achieved by tuning the 3-form fluxes, or by non-perturbative effects if 3-form fluxes preserve a discrete R-symmetry, e.g. $D3$ brane gaugino condensation which would give $w_0\sim e^{-bS_0}$, where S_0 is the vacuum value of the massive Type IIB dilaton.

Note that $b_i\Phi_0^i$ (no summation over i) have *universal* values at leading order in ϵ. In the normalization convention of Kähler moduli for which $\text{Re}(\Phi_0^i) \sim 1$, we have $b_i \sim \ln(M_{Pl}/m_{3/2})$, while K and their derivatives are generically of order unity.

The uplifting potential shifts the moduli vacuum values as

$$\delta\phi^I = -\frac{2\sqrt{2}}{3}\left(m_\phi^2\right)^{-1}_{IJ}(\partial_J K)V_{\text{lift}}, \qquad \delta a^I = 0. \tag{3.8}$$

As we will see, the moduli shifts $\delta\phi^I$ are all of $\mathcal{O}(\epsilon^2)$. Although tiny, this vacuum shift is the origin of nonzero vacuum values of F^I which were vanishing before the shift. On the other hand, the moduli masses at true vacuum $\langle\vec{\Phi}\rangle = \vec{\Phi}_0 + \delta\vec{\Phi}$ can be approximated well by the values at $\vec{\Phi}_0$ since the vacuum shift $\delta\vec{\Phi} = \mathcal{O}(\epsilon^2)$ gives a small correction to moduli masses.

For $\delta\vec{\Phi} = \mathcal{O}(\epsilon^2)$, the condition of vanishing cosmological constant requires

$$V_{\text{lift}}(\vec{\Phi}_0) \simeq 3|m_{3/2}|^2. \tag{3.9}$$

It is also straightforward to see that the moduli and axion masses are given by

$$\begin{aligned}
\left(m_\phi^2\right)_{ij} &= \mathcal{O}\left((m_{3/2}\ln(M_{Pl}/m_{3/2}))^2\right), \quad \left(m_\phi^2\right)_{TI} = 2m_{3/2}^2\partial_T\partial_I K, \\
\left(m_a^2\right)_{ij} &= \mathcal{O}\left((m_{3/2}\ln(M_{Pl}/m_{3/2}))^2\right), \quad \left(m_a^2\right)_{TI} = 0.
\end{aligned} \tag{3.10}$$

From this, one can easily finds $\delta\phi^I = (\delta s, \delta\phi^i)$ are all of the order of ϵ^2. The reason for $\delta\phi^i = \mathcal{O}(\epsilon^2)$ is that ϕ^i are heavy as $m_\phi^2 \sim m_{3/2}^2/\epsilon^2$. On the other hand, $m_s^2 \sim m_{3/2}^2$, thus the reason for $\delta s = \mathcal{O}(\epsilon^2)$ is different. Since both $\partial_T V_F$ and $\partial_T V_{\text{lift}}$ are vanishing at $\vec{\Phi}_0$, the uplifting potential does not directly induce a saxion tadpole. Rather, the saxion tadpole is induced by $\delta\phi^i$ through the Kähler mixing with ϕ^i. Explicitly, we find

$$\begin{aligned}
\delta\phi^i &= -\frac{2\sqrt{2}}{b_i\partial_i K}\sum_j \frac{1}{b_j}\left(\partial_i\partial_{\bar{j}}K - \frac{(\partial_i\partial_{\bar{T}}K)\partial_T\partial_{\bar{j}}K}{\partial_T\partial_{\bar{T}}K}\right), \\
\delta s &= -\frac{1}{\partial_T\partial_{\bar{T}}K}\sum_i \delta\phi^i\partial_T\partial_{\bar{i}}K,
\end{aligned} \tag{3.11}$$

for which

$$\begin{aligned}
F^i &= \frac{2m_{3/2}}{b_i} = (\Phi^i + \Phi^{i*})\frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}, \\
F^T &= -\sum_i \frac{\partial_i\partial_{\bar{T}}K}{\partial_T\partial_{\bar{T}}K}F^i = -\frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}\sum_i \frac{(\Phi^i + \Phi^{i*})\partial_i\partial_{\bar{T}}K}{\partial_T\partial_{\bar{T}}K},
\end{aligned} \tag{3.12}$$

at leading order in ϵ. Note that $\text{Re}(\Phi^i)$ are defined as the exponents of non-perturbative terms in KKLT superpotential $W = w_0 + \sum_i A_i e^{-b_i\Phi^i}$, thus correspond to the linear combinations of the 4-cycle volume moduli which obtain large positive vacuum values $\langle b_i\Phi^i\rangle \simeq \ln(M_{Pl}/m_{3/2}) \gg 1$ by non-perturbative superpotential. On the other hand, $\text{Re}(T)$ is a linear combination which can have any sign of vacuum value, even a vanishing vacuum value in some case such as the model (2.7). All of the above results are valid independently of the sign of the vacuum value of T. The F-components $F^i/(\Phi^i+\Phi^{i*})$ are universal at leading order in ϵ independently of the detailed form of the moduli Kähler potential, which is a characteristic feature of the KKLT-type moduli stabilization [41]. On the other hand, F^T appears to depend on the detailed form of K, particularly on $\partial_i\partial_{\bar{T}}K$. As we will see, if K takes a no-scale form, $F^T/(T+T^*)$ also becomes same as the universal $F^i/(\Phi^i+\Phi^{i*})$, thereby all moduli F-components (divided by moduli vacuum value) have universal values.

Combined with the moduli-axion kinetic term (3.5), the mass matrices (3.10) give the following pattern of moduli and axion mass eigenvalues:

$$\begin{aligned} &m_{\phi_i} \simeq m_{a_i} \sim m_{3/2}\ln(M_{Pl}/m_{3/2}), \\ &m_s = \sqrt{2}m_{3/2}, \quad m_a = 0, \end{aligned} \tag{3.13}$$

where the mass eigenstate saxion and axion are mostly $\mathrm{Re}(T)$ and $\mathrm{Im}(T)$, respectively. The SUSY-breaking F-components of all moduli are of the order of $m_{3/2}/4\pi^2$:

$$\frac{F^I}{\Phi^I+\Phi^{I*}} \sim \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})} \sim \frac{m_{3/2}}{4\pi^2}. \tag{3.14}$$

As a result, the soft SUSY breaking terms of visible fields receive comparable contributions from moduli mediation and anomaly mediation [16], leading to the mirage mediation pattern of superparticle masses discussed in [20, 21, 22, 23]. We also find the modulino and axino masses are given by

$$m_{\tilde{\phi}_i} \sim m_{3/2}\ln(M_{Pl}/m_{3/2}), \quad m_{\tilde{a}} = m_{3/2}. \tag{3.15}$$

Note that the helicity 1/2 component of gravitino mostly comes from the Goldstino localized on the SUSY breaking brane, not from the Kähler modulino/axino.

We stress that the above results of moduli/modulino masses and F-components are obtained for generic effective SUGRA under the assumptions of (i) the axionic shift symmetries

$$U_{PQ}: T \to T + i\beta, \quad U(1)_i: \Phi^i \to \Phi^i + i\beta_i, \tag{3.16}$$

broken dominantly by non-perturbative superpotential except for $U(1)_{PQ}$, and (ii) a sequestered uplifting potential. In compactified string theory, the pseudoscalar components of Kähler moduli correspond to the zero modes of antisymmetric tensor gauge field, and thus the axinonic shift symmetries broken only by non-perturbative effects are generic feature of 4D effective theory. Also, as we have noticed, a sequestered uplifting potential is the most plausible form of uplifting mechanism in string compactification with warped throat. It arises from generic SUSY-breaking brane stabilized at the IR end of warped throat. Thus the moduli mass pattern (3.13) and the SUSY-breaking F-components (3.14) can be considered as a quite robust prediction of $U(1)_{PQ}$-invariant generalization of KKLT moduli stabilization giving the QCD axion.

Now, let us note that the Kähler moduli F-components become *universal* for the no-scale Kähler potential satisfying

$$\sum_J K^{I\bar{J}}\partial_{\bar{J}}K = -(\Phi^I + \Phi^{I*}). \tag{3.17}$$

Indeed, at leading order in α' and string loop expansion, the Kähler potential of Kähler moduli in Type IIB string compactification takes the no-scale form. Then for SUSY configuration satisfying $\partial_T K = 0$, the above relation gives

$$-\partial_{\bar{T}}K = \sum_I(\Phi^I + \Phi^{I*})K_{I\bar{T}} = (T+T^*)\partial_T\partial_{\bar{T}}K + \sum_i(\Phi^i + \Phi^{i*})\partial_i\partial_{\bar{T}}K = 0 \tag{3.18}$$

Applying this to (3.12), we find

$$\frac{F^T}{T+T^*} = \frac{F^i}{\Phi^i + \Phi^{i*}} = \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}, \tag{3.19}$$

i.e. all Kähler moduli including T have universal values of F^I/Φ^I although T and Φ^i are stabilized by different mechanisms. As we will see, this universality of F^I/Φ^I has an interesting implication for the flavor conservation of soft terms.

Much of the low energy properties of the QCD axion is determined by the axion decay constant $v_{\rm PQ}$ which is defined through the effective coupling between the axion and the gluon anomaly:

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{4g^2_{QCD}}F^{a\mu\nu}F^a_{\mu\nu} + \frac{1}{32\pi^2}\frac{a}{v_{\rm PQ}}F^{a\mu\nu}\tilde{F}^a_{\mu\nu}, \tag{3.20}$$

where $\tilde{F}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^a_{\rho\sigma}$. Using that a is contained mostly in $\mathrm{Im}(T)$, for the QCD gauge kinetic function $f_{\rm QCD}$ which takes the form of (2.1), we find

$$v_{\rm PQ} = \frac{M_{Pl}}{8\pi^2}\frac{(\partial_T\partial_{\bar{T}}K)^{1/2}}{\partial_T[\ln \mathrm{Re}(f_{\rm QCD})]}, \tag{3.21}$$

where $M_{Pl} \simeq 2\times10^{18}$ GeV. As the above result shows, the precise value of $v_{\rm PQ}$ is somewhat model-dependent, but generically around 10^{16} GeV.

Such a large value of $v_{\rm PQ}$ might cause the cosmological problem that the cosmological axion density produced by initial misalignment overcloses the Universe [42]. Interestingly, for the moduli stabilization scenario under consideration, this cosmological axion problem can be significantly ameliorated by the late decay of saxion [43] which has a right mass to decay right before the big-bang nucleosynthesis (BBN) for the most interesting case that the visible sector superparticle masses are of the order of the weak scale. As will be discussed in the next section, the soft SUSY breaking masses of visible fields are given by $m_{\rm soft} \sim \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}$. As a result, the saxion mass $m_s \simeq \sqrt{2}m_{3/2} \sim 50$ TeV and $m_{\phi_i} \simeq m_{a_i} \sim m_{3/2}\ln(M_{Pl}/m_{3/2}) \sim 10^3$ TeV for $m_{\rm soft} \sim 1$ TeV. On the other hand, moduli (including the saxion) couplings to visible fields are suppressed by $1/M_{Pl} \sim 1/8\pi^2 v_{\rm PQ}$, giving the reheat temperature after saxion decay $T_{RH} \sim 6$ MeV for $m_s \sim 50$ TeV. If the early Universe before the saxion decay were dominated by the coherent oscillation of saxion field, such late decay of saxions dilutes the axion density as well as the potentially dangerous primordial gravitinos, therefore might allow the model to avoid the cosmological gravitino, moduli and axion problems [43]. In particular, in this scenario, the QCD axion can be a good dark matter candidate under a mild assumption on the initial axion misalignment although the axion decay constant is around the GUT scale. A detailed analysis of the moduli and axion cosmology in the generalized KKLT set up with QCD axion will be presented elsewhere [27].

4. Flavor and CP conserving soft terms

In this section, we discuss the soft terms in more detail, focusing on the CP and flavor issues. To this end, let us include the visible gauge and matter superfields in the effective SUGRA action:

$$\begin{aligned} &\int d^4\theta CC^* \left[-3\exp\left\{-\frac{1}{3}\Big(K_0 + Z_p Q^p Q^{p*}\Big)\right\} - CC^*\Lambda^2\bar{\Lambda}^2 e^{4A}\mathcal{P}_0\right] \\ &+ \left[\int d^2\theta \left\{\frac{1}{4} f_a W^{a\alpha} W^a_\alpha + C^3\Big(w_0 + \sum_i A_i e^{-b_i\Phi^i} + \frac{1}{6}\lambda_{pqr} Q^p Q^q Q^r\Big)\right\} + \text{h.c.}\right], \end{aligned} \tag{4.1}$$

where the axionic shift symmetries (3.16) require that moduli Kähler potential K_0, matter Kähler metric Z_p, and the holomorphic gauge kinetic functions are given by

$$\begin{aligned} K_0 &= K_0(\Phi^I + \Phi^{I*}), \quad Z_p = Z_p(\Phi^I + \Phi^{I*}), \\ f_a &= \sum_I c_I \Phi^I = c_T T + \sum_i c_i \Phi^i, \end{aligned} \tag{4.2}$$

where c_I are real constants with nonzero c_T. The soft SUSY breaking terms of canonically normalized visible fields can be written as

$$\mathcal{L}_{\rm soft} = -\frac{1}{2} M_a \lambda^a \lambda^a - \frac{1}{2} m_r^2 |\tilde{Q}^r|^2 - \frac{1}{6} A_{pqr} y_{pqr} \tilde{Q}^p \tilde{Q}^q \tilde{Q}^r + \text{h.c.}, \tag{4.3}$$

where λ^a are gauginos, $\tilde{Q}^r$ are the scalar component of Q^r and y_{pqr} are the canonically normalized Yukawa couplings:

$$y_{pqr} = \frac{\lambda_{pqr}}{\sqrt{e^{-K_0} Z_p Z_q Z_r}}. \tag{4.4}$$

For $F^I/\Phi^I \sim m_{3/2}/4\pi^2$, the soft parameters at energy scale just below $M_{GUT} \sim 2 \times 10^{16}$ GeV are determined by the modulus-mediated and anomaly-mediated contributions which are comparable to each other. One then finds [16]

$$\begin{aligned} M_a &= \tilde{M}_a + \frac{m_{3/2}}{16\pi^2} b_a g_a^2, \\ A_{pqr} &= \tilde{A}_{pqr} - \frac{m_{3/2}}{16\pi^2}(\gamma_p + \gamma_q + \gamma_r), \\ m_r^2 &= \tilde{m}_r^2 - \frac{1}{16\pi^2}(m_{3/2}^*\Theta_r + m_{3/2}\Theta_r^*) - \left|\frac{m_{3/2}}{16\pi^2}\right|^2 \dot{\gamma}_i, \end{aligned} \tag{4.5}$$

where the moduli-mediated contributions are given by

$$\begin{aligned}
\tilde{M}_a &= \sum_I F^I \partial_I \ln \mathrm{Re}(f_a), \\
\tilde{m}_r^2 &= -\sum_{IJ} F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} \ln(e^{-K_0/3} Z_r), \\
\tilde{A}_{pqr} &= -\sum_I F^I \partial_I \ln \left(\frac{\lambda_{pqr}}{e^{-K_0} Z_p Z_q Z_r} \right) = \sum_I F^I \partial_I \ln(e^{-K_0} Z_p Z_q Z_r).
\end{aligned} \tag{4.6}$$

Here we have used that the holomorphic Yukawa couplings λ_{pqr} are independent of Φ^I as a consequence of the axionic shift symmetries. The one-loop beta function coefficient b_a, the anomalous dimension γ_p and its derivative $\dot{\gamma}_p$, and Θ_p are defined as

$$\begin{aligned}
b_a &= -3\mathrm{tr}\left(T_a^2(\mathrm{Adj})\right) + \sum_p \mathrm{tr}\left(T_a^2(Q^p)\right), \\
\gamma_p &= 2\sum_a C_2^a(Q^p) g_a^2 - \frac{1}{2}\sum_{qr} |y_{pqr}|^2, \\
\dot{\gamma}_p &= 8\pi^2 \frac{d\gamma_p}{d\ln\mu}, \\
\Theta_p &= 2\sum_a C_2^a(Q^p) g_a^2 \tilde{M}_a - \frac{1}{2}\sum_{qr} |y_{pqr}|^2 \tilde{A}_{pqr},
\end{aligned} \tag{4.7}$$

where the quadratic Casimir $C_2^a(Q^p) = (N^2-1)/2N$ for a fundamental representation Q^p of the gauge group $SU(N)$, $C_2^a(Q^p) = q_p^2$ for the $U(1)$ charge q_p of Q^p, and $\omega_{pq} = \sum_{rs} y_{prs} y_{qrs}^*$ is assumed to be diagonal.

As was noticed before [37], the axionic shift symmetries (3.16) assure that the above soft terms preserve CP. To see this, let us first note that $\partial_I K_0$, $\partial_I Z_p$, $\partial_I V_{\mathrm{lift}}$, $\partial_I \lambda_{pqr}$, and $\partial_I f_a$ are all real as a consequence of the axionic shift symmetries. Combined with $U(1)_R$ transformation, the axionic shift symmetries also allow that w_0 and A_i in the moduli superpotential are chosen to be real without loss of generality, leading to real $m_{3/2}$ and F^I. Obviously then all soft parameters are real, thus preserve CP *independently of* the detailed forms of the moduli Kähler potential, matter Kähler metric and the gauge kinetic functions as long as they respect the axionic shift symmetries.

Recently, it has been noticed that $e^{-K_0/3} Z_p$ have a definite scaling property under the overall rescaling of the CY metric, $g_{mn} \to \lambda g_{mn}$, at leading order in α' and string loop expansion [44]. Under this rescaling of metric, the CY volume and Kähler moduli transform as

$$V_{CY} \to \lambda^3 V_{CY}, \qquad \Phi^I \to \lambda^2 \Phi^I, \tag{4.8}$$

while the IIB dilaton and complex structure moduli do not transform. The normalized wavefunction of matter zero mode Q_p transforms as

$$\psi_p \to \lambda^{-d_p/4} \psi_p \tag{4.9}$$

under the metric rescaling, where d_p is the internal dimension of the subspace σ_p over which Q_p can propagate. The physical Yukawa couplings are then given by the integral of matter wavefunctions over a subspace σ_{pqr} of the intersection of σ_p, σ_q and σ_r:

$$y_{pqr} = \int_{\sigma_{pqr}} dx^{d_{pqr}} \sqrt{g} \psi_p \psi_q \psi_r \tag{4.10}$$

which transforms as

$$y_{pqr} \to \lambda^{(2d_{pqr}-d_q-d_q-d_r)/4} y_{pqr}, \tag{4.11}$$

where d_{pqr} is the dimension of σ_{pqr}. On the other hand, the holomorphic Yukawa couplings λ_{pqr} in 4D effective SUGRA do not transform under the metric rescaling since they are independent of Kähler moduli due to the axionic shift symmetries (3.16). Then, to match with the scaling property of y_{pqr} which is given by $y_{pqr} = \lambda_{pqr}/\sqrt{e^{-K_0} Z_p Z_q Z_r}$ in 4D effective SUGRA, the matter Kähler metric Z_p should transform as

$$Z_p(\lambda^2(\Phi^I + \Phi^{I*})) = \lambda^{2(n_p-1)} Z_p(\Phi^I + \Phi^{I*}), \tag{4.12}$$

where the scaling weights n_p satisfy

$$4(n_p + n_q + n_r) = d_p + d_q + d_r - 2d_{pqr} \tag{4.13}$$

for the combinations of Q_p with nonzero Yukawa coupling. Here we have used that the leading order Káhler potential of Kähler moduli is given by $K_0 = -2\ln(V_{CY})$.

Combined with the universality of F^I/Φ^I obtained for a no-scale form of moduli Kähler potential, the above scaling property of the matter Kähler metric assures that soft terms derived from our moduli stabilization set-up preserve flavor. For the universal F-components:

$$\frac{F^I}{\Phi^I + \Phi^{I*}} = \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})} \equiv M_0, \tag{4.14}$$

the Kähler potential and gauge kinetic functions of (4.2) give

$$\begin{aligned}
\tilde{M}_a &= M_0 \sum_I (\Phi^I + \Phi^{I*}) \partial_I \ln \mathrm{Re}(f_a) = M_0, \\
\tilde{m}_r^2 &= -M_0^2 \sum_{IJ} (\Phi^I + \Phi^{I*})(\Phi^J + \Phi^{J*}) \partial_I \partial_{\bar{J}} \ln(e^{-K_0/3} Z_r), \\
\tilde{A}_{pqr} &= M_0 \sum_I (\Phi^I + \Phi^{I*}) \partial_I \ln(e^{-K_0} Z_p Z_q Z_r).
\end{aligned} \tag{4.15}$$

Then the scaling property of $e^{-K_0/3} Z_p$ leads to

$$\tilde{m}_r^2 = n_r M_0^2, \quad \tilde{A}_{pqr} = (n_p + n_q + n_r) M_0, \tag{4.16}$$

i.e. the moduli-mediated soft scalar masses and A-parameters are determined simply by the matter scaling weights. It is highly plausible that matter fields with the same gauge quantum numbers have a common geometric origin, therefore have the same scaling weights.

Then the soft masses take a phenomenologically desirable *flavor-blind* form independently of the detailed form of the matter and moduli Kähler potential at leading order in α' and string loop expansion.

Taking into account the 1-loop RG evolution, the soft masses of (4.5) at $M_{GUT} \sim 2 \times 10^{16}$ GeV lead to low energy soft masses described by the mirage messenger scale [20]:

$$M_{\rm mir} \sim \frac{M_{GUT}}{(M_{Pl}/m_{3/2})^{1/2}} \sim 3 \times 10^9 \text{ GeV}. \tag{4.17}$$

The low energy gaugino masses are given by

$$M_a(\mu) = M_0 \left[1 - \frac{1}{8\pi^2}\, b_a g_a^2(\mu) \ln\left(\frac{M_{\rm mir}}{\mu}\right) \right] = \frac{g_a^2(\mu)}{g_a^2(M_{\rm mir})} M_0, \tag{4.18}$$

showing that the gaugino masses are unified at $M_{\rm mir}$, while the gauge couplings are unified at M_{GUT}. The low energy values of A_{pqr} and m_r^2 generically depend on the associated Yukawa couplings y_{pqr}. However if y_{pqr} are small enough, e.g. the case of the first and second generations of quarks and leptons, or

$$n_p + n_q + n_r = 1 \quad \text{for} \quad y_{pqr} \sim 1, \tag{4.19}$$

their low energy values are given by [20]

$$\begin{aligned}
A_{pqr}(\mu) &= M_0 \left[n_p + n_q + n_r + \frac{1}{8\pi^2}(\gamma_p(\mu) + \gamma_q(\mu) + \gamma_r(\mu)) \ln\left(\frac{M_{\rm mir}}{\mu}\right) \right], \\
m_r^2(\mu) &= M_0^2 \left[n_r - \frac{1}{8\pi^2} Y_r \left(\sum_p n_p Y_p \right) g_Y^2(\mu) \ln\left(\frac{M_{GUT}}{\mu}\right) \right. \\
&\qquad \left. + \frac{1}{4\pi^2} \left\{ \gamma_r(\mu) - \frac{1}{2}\frac{d\gamma_r(\mu)}{d\ln\mu} \ln\left(\frac{M_{\rm mir}}{\mu}\right) \right\} \ln\left(\frac{M_{\rm mir}}{\mu}\right) \right],
\end{aligned} \tag{4.20}$$

where Y_p is the $U(1)_Y$ charge of Q^p. In this case, the A-parameters and sfermion masses at $M_{\rm mir}$ are (approximately) same** as the moduli-mediated contributions at M_{GUT}. If the squarks and sleptons have common scaling weight, which is a rather plausible possibility, the squark and slepton masses appear to be unified at $M_{\rm mir}$ for the case that either the associated Yukawa couplings are small or the scaling weights obey the condition (4.19). We note that (4.19) is obtained when $d_p + d_q + d_r - 2d_{pqr} = 4$, for instance when y_{pqr} is given by an integral of the quark/lepton and Higgs wavefunctions over 4-cycle, for which $d_p = d_q = d_r = d_{pqr} = 4$, or when y_{pqr} is given by an integral over 2-cycle of the quarks/leptons wavefunctions defined on 2-cycle and the Higgs wavefunction defined on 4-cycle, for which $d_{pqr} = 2$, $d_{\rm quark} = d_{\rm lepton} = 2$, and $d_{\rm Higgs} = 4$.

So far, we have discussed the soft terms in generic $U(1)_{PQ}$-invariant generalization of KKLT moduli stabilization which can give the QCD axion solving the strong CP problem. The soft terms preserve CP due to the axionic shift symmetries broken by non-perturbative

**Note that $\sum_p n_p Y_p = 0$ if the scaling weights n_p obey the $SU(5)$ relation. Even when $\sum_p n_p Y_p \neq 0$, the part of $m_r^2(M_{\rm mir})$ due to nonzero $\sum_p n_p Y_p$ is numerically negligible.

superpotential [37]. At leading order in α' and string loop expansion, the moduli F-components F^I/Φ^I have universal vacuum values and also the matter Kähler metrics have a definite scaling property under the overall rescaling of CY metric. As a consequence, the moduli-mediated contributions to soft terms take a highly predictive form, i.e. the moduli-mediated gaugino masses at M_{GUT} are given by $\tilde{M}_a = M_0 \equiv m_{3/2}/\ln(M_{Pl}/m_{3/2})$ and the moduli-mediated A-parameters and sfermion masss at M_{GUT} are simply determined by the scaling weights as (4.16), independently of the detailed forms of the moduli Kähler potential and matter Kähler metric. Since the matter fields with the same gauge quantum numbers are expected to have common scaling weight, the soft terms in our moduli stabilization set-up naturally preserve flavor at leading order in α' and string loop expansion . If the higher order corrections[††] in underlying string compactification can be made to be small enough, this flavor-universality at leading order approximation would ensure that the model naturally passes the constraints from low energy flavor violation.

5. Conclusion

The QCD axion provides an attractive solution to the strong CP problem. As it contains numerous axions, string theory is perhaps the most plausible framework to give the QCD axion. In supersymmetric compactification, the QCD axion accompanies its scalar partner, the saxion, which should be stabilized while keeping the QCD axion as a flat direction until the low energy QCD instanton effects are taken into account. In this paper, we show that a simple generalization of KKLT moduli stabilization with the number of Kähler moduli $h_{1,1} > 1$ provides such saxion stabilization set-up.

Quite interestingly, although some details of moduli stabilization are different from the original KKLT scenario, the resulting soft SUSY breaking terms still receive comparable contributions from moduli mediation (including the saxion mediation) and anomaly mediation, therefore take the mirage mediation pattern with $m_{\rm soft} \sim \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}$ as in the KKLT case. Furthermore, the soft terms naturally preserve flavor and CP as a consequence of approximate scaling and axionic-shift symmetries of the underlying string compactification, independently of the detailed forms of moduli Kähler potential and matter Kähler metric. As for the moduli spectrum, saxion has a mass $m_s \simeq \sqrt{2} m_{3/2}$, while other moduli (except for the QCD axion) have a mass of the order of $m_{3/2}\ln(M_{Pl}/m_{3/2})$ or heavier, independently of the detailed form of the moduli Kähler potential. This pattern of moduli masses might avoid the cosmological gravitino, moduli and axion problems for the most interesting case that $m_{\rm soft} \sim 1$ TeV since the saxion has a right mass to decay right before the big-bang nucleosynthesis. In particular, it might allow the QCD axion to be a good dark matter candidate under a mild assumption on the initial axion misalignment although the axion decay constant is near the GUT scale.

[††] An example of potentially important higher order correction would be the effect of magnetic flux $\mathcal{F}$ on $D7$ brane wrapping a 4-cycle σ, whose strength is controlled by $\frac{\alpha'^2}{8\pi^2}\int_\sigma \mathcal{F}\wedge\mathcal{F}$.

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